STOCHASTIC PRODUCTION FRONTIER: FRAMEWORK AND DEVELOPMENT

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Abstract
This paper explores the framework and development of stochastic frontier Approach (SFA). The original idea of the SFA and its theoretical framework is discussed to provide a basic foundation of the approach. The development of SFA with more flexible distribution assumptions follows the pioneering model. Experts also develop the time-variant technical efficiency models, in order to allow variation between times for a production unit. The most recent development is the panel data SFA, which includes the two-stage and the one-stage procedures.

Keywords: Stochastic Frontier Approach, Time-variant Technical Efficiency, Panel data.

An Overview of the Stochastic Frontier Approach

The conventional SFA can be tracked back to two pioneering papers, published nearly simultaneously by two teams: Aigner *et al.* (1977) and Meeusen and van den Broeck (1977). These two papers propose a common structure of two-part composed error, developed under a stochastic production frontier framework. The first part error accounts for random statistical noise representing factors such as weather, luck, measurement errors, and other unpredictable aspects outside a firm's control. The second part error is intended to capture the technical inefficiency of firms.

The typical functional form of the SFA, as proposed by the two pioneering papers, can be written as:

$$Y_i = f(\mathbf{X}_i;\alpha_0,\boldsymbol{\beta}).\exp(v_i - u_i) \qquad (1)$$

where Y_i is the scalar output of firm i ($i=1,2,\ldots,N$),

$f(\mathbf{X}_i;\alpha_0,\boldsymbol{\beta}).\exp(v_i)$ is the stochastic production frontier, $\mathbf{X}_i$ is a *(1xk)* vector of inputs used by firm *i*,

$\boldsymbol{\beta}$ is a *(kx1)* vector of slope parameters,

α_0 is production frontier intercept,

$\exp(v_i - u_i)$ is the combined error term,

v_i is a two-sided random statistical noise of firm *i*, with *iid* $N\left(0,\sigma_v^2\right)$

u_i is one-side error component representing technical inefficiency.

In a linear format for firm *i*, Equation (1) can be expressed as

$$y_i = \alpha_0 + \mathbf{x}_i\boldsymbol{\beta} + v_i - u_i \qquad (2)$$

or

$$y_i = \alpha_0 + \begin{bmatrix} x_{1i} & x_{2i} & . & . & . & x_{ki} \end{bmatrix} \begin{bmatrix} {}_1 \\ {}_2 \\ . \\ . \\ . \\ {}_k \end{bmatrix} + v_i - u_i \qquad (3)$$

where y_i is the scalar of the logarithm of output for firm *i* ($i=1,2,\ldots,N$), $\mathbf{x}_i$ is a *(1xk)* vector of the logarithm of inputs used by firm *i*, and other variables are as previously defined.

The basic idea behind the SFA model, as shown in Equation (1), comes from the difference between the assumption in a conventional production function and the observed firms' outputs. The conventional production function specifies the maximum possible output levels from a given set of inputs (i.e., firms are assumed to be producing at the full efficiency level), whereas the observed output data are smaller than or equal to the maximum possible output (i.e., some firms are producing below the full efficiency level). Thus, technical inefficiencies exist in firms' production. Incorporating the technical efficiency, the SFA introduces a one-side error term, ui. Hence, the objective of the SFA is not only estimating the parameters of production technology , as in the conventional production function, but also measuring the technical inefficiency by separating the two error components (ui and vi).

The pioneering papers of Aigner *et al.* (1977) and Meeusen and van den Broeck (1977) propose the maximum-likelihood (ML) method to achieve the objectives of the SFA. This method requires a distributional assumption for the two error components (v_i and u_i) and an assumption of non-correlation between the one-side error term (u_i) and input variables (x_i). Given these assumptions, the early stochastic frontier models are intended for cross-sectional applications. In dealing with the distributional assumption, Aigner *et al.* (1977) suggest normal and half-normal distributions for v_i and u_i, respectively. Meeusen and van den Broeck (1977), on the other hand, propose normal and exponential distributions.

The Development of Distributional Assumption

Following the two pioneering papers, subsequent researchers develop more flexible form of distributions. Greene (1980), for example, suggests normal and gamma distributions by introducing additional parameters to be estimated, which provides a more flexible representation of the pattern of technical inefficiency in the data. Similarly, Stevenson (1980) proposes normal and truncated-normal distributions by allowing the normal distribution, which is truncated below at zero, to have a non-zero mode.[1]

The availability of various distributional assumptions, as proposed above, leads researchers to question whether the distributional assumption significantly affects the measurement of technical efficiency. The mean of technical efficiency scores tend to be sensitive on the distributional assumption, as shown by Greene (1990). However, neither the ranking of firms by their technical efficiency scores nor the deciles composition of efficiency scores seems to be sensitive to the distributional assumptions.

Kumbhakar and Lovell (2000), for example, show a very close concordance of the ranking of technical efficiency scores from separate estimation results using those four different distributional assumptions mentioned above. Similarly, Horrace (2005) find that the ranking of firms based on their technical efficiency scores do not change when the four different distributional assumptions are applied interchangeably. Findings from these two studies provide support for Ritter and Simar's (1999) argument that the choice between alternative distributional assumptions is of little consequence on the measurement of technical inefficiency. From what follows, the practical evidence indicates that the choice between alternative distributional assumptions is largely immaterial. Nevertheless, the two original distributional assumptions remain as the favorable options for the vast majority of empirical studies (Kumbhakar and Lovell, 2000). The earlier empirical papers adopting the original distributions include Kalirajan (1981; 1982; 1989), Kalirajan and Flinn (1983), Kalirajan and Shand (1986), and Pitt and Lee (1981).

The distributional assumptions of the technical efficiency might be important for cross-sectional data. However, more recent literature on stochastic frontier models in the context of panel data has relaxed these strong distributional assumptions. The repeated observations over time for a given firm in

[1] Kumbhakar and Lovell (2000) provide an excellent discussion on the distributional assumptions of SFAs.

panel data context can serve as a substitute for the distributional assumptions (Lee, 2006). With the repeated observation overtime, the estimates of technical efficiency under panel data context provide more desirable statistical properties. As argued by Schmidt and Sickles (1984), panel data facilitates a more accurate measure of technical efficiency (ui), when it is separated from the stochastic noise at the level of individual firm (vi).

Applications of SFA on panel data are first introduced by Pitt and Lee (1981) and Schmidt and Sickles (1984). In their papers, Pitt and Lee (1981) extend the cross-sectional stochastic frontier model to a panel data context under ML estimation, while Schmidt and Sickles (1984) apply fixed-effect and random-effect panel data on SFA. Subsequently, Kumbhakar (1987) and Battese and Coelli (1988) extend Pitt and Lee's (1981) model by focusing on a more general distribution of technical inefficiency. The functional form of these early panel data stochastic frontiers can be written as:

$$Y_{it} = f(\mathbf{X}_{it}; \sim, \boldsymbol{\beta}).\exp(v_{it} - u_i) \qquad (4)$$

Compared to the original stochastic frontier model in equation (1), the stochastic frontier model in equation (4) has an additional subscript *t* for explaining time. This additional *t* reflects that the data are panel in nature, with a cross-sectional dimension of *i=(1, 2, ..., N)* and a time dimension of *t = (1, 2, ..., T)*. In a linear format for firm *i* at time *t*, the equation (4) is expressed as:

$$\begin{aligned} y_{it} &= \sim_0 + \mathbf{x}_{it}\boldsymbol{\beta} + v_{it} - u_i \\ &= \sim_i + \mathbf{x}_{it}\boldsymbol{\beta} + v_{it} \end{aligned} \qquad (5)$$

or

$$y_{it} = \sim_i + \begin{bmatrix} x_{1it} & x_{2it} & . & . & . & x_{kit} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ . \\ . \\ . \\ k \end{bmatrix} + v_{it} \qquad (6)$$

where y_{it} is the scalar of the logarithm of output for firm *i (i=1,2,...,N)* at time *t (t=1,2,...,T)*, $\mathbf{x}_{it}$ is a *(1xk)* vector of the logarithm of inputs used by firm *i* at time *t*, $\boldsymbol{\beta}$ is a *(kx1)* vector of unknown parameters, $\sim_i = \sim_0 - u_i$ is the intercept for firm *i* that is invariant at all time *t*.

Time-Variant Technical Efficiency

Equation (5) shows that the early models of panel data SFA assume time-invariant technical efficiency. This assumption is very strong, especially for firms operating under a competitive environment. Technical efficiency scores are expected to change through time if firms compete in a market. Therefore, more recent literature on panel data SFA focuses on relaxing this strong assumption. Scholars introduce a stochastic frontier model with time-varying technical efficiency for panel data.

There are four seminal papers on SFA showing that the time-invariant assumption for technical efficiency (TE) could be relaxed: Cornwell et al. (1990, hereafter CSS), Kumbhakar (1990), Battese and Coelli (1992, hereafter BC), and Lee and Schmidt (1993, hereafter LS). These four papers can be divided into two groups based on the methods of estimation. CSS and LS follow traditional panel data methods and Kumbhakar and BC employ ML methods. Generally, the SFA model with time-varying TE is written as:

$$\begin{aligned} y_{it} &= \sim_{0t} + \mathbf{x}_{it}\boldsymbol{\beta} + v_{it} - u_{it} \\ &= \sim_{it} + \mathbf{x}_{it}\boldsymbol{\beta} + v_{it} \end{aligned} \qquad (7)$$

where $\sim_{0t}$ is the production frontier intercept common to all firms in time *t*, $\sim_{it} = \sim_{0t} - u_{it}$ is the intercept for firm *i (i=1,2,...,I)* that varies through time *t (t=1,2,...,T)*. Note that in equation (7), the technical efficiency components, *u*, has an additional subscript *t* that reflects the time-varying TE.

Given a $(N \times T)$ panel in the SFA model in equation (7), it is not possible to obtain estimates of $(N \times T)$ intercepts $\sim_{it}$ and the slope of vector parameters β. In addressing this problem, Cornwell *et al.* (1990) specifies $\sim_{it}$ as:

$$\sim_{it} = \Omega_{i0} + \Omega_{i1}t + \Omega_{i2}t^2 \quad (8)$$

where $[\Omega_{i0}, \Omega_{i1}, \Omega_{i2}]$ are parameters to be estimated. Note that, under this specification, the number of intercept parameters is reduced to $(N \times [\Omega_{i0}, \Omega_{i1}, \Omega_{i2}])$ or $(N \times 3)$, which is possible to estimate. This specification is useful, particularly for a panel with a small numbers of cross-sections. However, in a practical sense, it will be burdensome if the number of cross-sections is large.

Lee and Schmidt (1993), on the other hand, specifies the time-varying TE into

$$u_{it} = \Omega_t u_i \quad (9)$$

for $\Omega_t = [\Omega_1, \Omega_2, ..., \Omega_T]$ represents a set of time dummy variables. By normalizing $\Omega_1 = 1$, Lee and Schmidt (1993) shows that the number of intercept parameters reduce to $(T-1)$. If compared to Cornwell *et al.* (1990), the specification of Lee and Schmidt (1993) has an advantage in terms of flexibility in the pattern of TE over time, but has a disadvantage in the sense that it imposes a common time path of variation on TE for all firms. The Lee and Schmidt's model is useful for panel data with a short time series.

Under a different method of estimation, Kumbhakar (1990) proposes a SFA model with time-varying TE as a parametric function of time. The time-varying TE for this model can be written as

$$u_{it} = \ (t).u_i$$

$$(t) = \left[1 + \exp\left\{^{-}t + ¢t^2\right\}\right]^{-1} \quad (10)$$

where θ and δ are two additional unknown parameters to be estimated, and u_i is assumed to have a half-normal distribution. The function (t) has a value between zero and one, which can increase or decrease monotonically. Kumbhakar's model, as written in equation (10), shows that there are only two additional parameters (¥ and ¢) to be estimated under a ML method.

Also using a ML method, Battese and Coelli (1992) suggest an alternative to the Kumbhakar (1990) model. They propose time-varying TE under a different function of time, which can be defined as

$$u_{it} = \exp\left[-¦\left(t - T\right)\right].u_i \quad (11)$$

where ¦ is an unknown parameter to be estimated, which has a value between zero and one, and u_i is assumed to have a truncated-normal distribution. To solve the ML estimation, Battese and Coelli (1992) replace the common variance of error components $(\pm_v^2 \text{ and } \pm_u^2)$ with

$$\pm^2 = \pm_v^2 + \pm_u^2 \text{ and } ¥ = \frac{\pm_u^2}{\left(\pm_u^2 + \pm_v^2\right)}.$$

The Battese and Coelli (1992) model has advantages in that it has only one additional unknown parameter (¦) and it is applicable on unbalanced panel data. The disadvantage is mostly related to an assumption of a monotonic increase and decrease in TE over time, which is particularly severe under panel data with a large time dimension.

Cuesta (2000) and Orea (2002) extend Battese and Coelli's (1992) model by relaxing the assumption of monotonic increase and decrease in TE over time. Cuesto (2000) proposes a time-varying TE, which can be expressed as

$$u_{it} = \exp\left[-¦_i\left(t - T\right)\right].u_i \quad (12)$$

In this model, Cuesto replaces ¦ with $¦_i$, which shows that each individual firm has its own temporal pattern of TE. Hence, the parameters to be estimated now increase from one to the number of cross-sections *(i=1,2,...,N)*. Similarly to Cornwell *et al.* (1990), Cuesta's (2000) model has a disadvantage when dealing with panel data with a large cross-sectional observation.

On the other hand, Orea (2002) suggests a time-varying TE as

$$u_{it} = \exp\left[-\eta_1 (t-T) - \eta_2 (t-T)^2 \right] . u_i \quad (13)$$

Orea (2002) adds an additional parameter η_2 into Battese and Coelli's (1992) model to relax the monotonic temporal pattern of TE. In Orea's model, the numbers of unknown parameters associated with TE increases to two $(\eta_1 \text{ and } \eta_2)$, rather than one (η).

The Panel Data SFA with Exogenous Effects on TE

The recently developed SFA for panel data has focused on exogenous variables, which may affect a firm's productivity performance. These exogenous variables are neither inputs for production nor output from production, but they are more related to the environment in which the production occurs. Such variables can be the age of firms, size of firms, degree of competition, managerial characteristics, input and output quality, and so on. A way to incorporate these variables into the SFA model is by including them as exogenous variables affecting technical inefficiency. By doing so, this recently developed SFA is intended to show that a firm's productivity performance depends not only on the quantity of inputs and outputs but also on a firm's specific characteristics.

The panel data SFA with exogenous variables on TE can be written in a general form as

$$y_{it} = \alpha_{0t} + \mathbf{x}_{it}\boldsymbol{\beta} + v_{it} - u_{it} \quad (14a)$$

$$u_{it} = \mathbf{z}_{it}\boldsymbol{\gamma} + \varepsilon_{it} \quad (14b)$$

where z is a *(1xm)* vector of explanatory variables affecting technical inefficiency of production, $\boldsymbol{\gamma}$ is a *(mx1)* vector of parameters of technical inefficiency function, and ε is a random variable. The inefficiency function in equation (14b) can also be written as

$$u_{it} = \left[z_{1it} \quad z_{2it} \quad . \quad . \quad . \quad z_{mit} \right] \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ . \\ . \\ . \\ \gamma_m \end{bmatrix} + \varepsilon_{it} \quad (15)$$

Survey studies, such as Kumbhakar and Lovell (2000) and Coelli *et al.* (2005), show that this stream of SFA can be divided into two groups. The first group is the early two-stage approach and the second group is the more recent one-stage approach.

The early two-stage approach for incorporating exogenous variables into productivity performance is first proposed by Kalirajan (1981) and Pitt and Lee (1981). In the first stage, this group of SFA estimates production frontier, as in equation (14a), and measures the technical efficiency index of each individual firm. In the second stage, the obtained technical efficiency index is regressed against a set of exogenous variables, as in equation (14b), using the standard OLS method. This two-stage approach assumes that the exogenous variables indirectly affect output through their effect on technical inefficiency. Empirical papers applying this two-stage approach include Kalirajan (1982; 1989), Kalirajan and Flinn (1983), Kalirajan and Shand (1986; 1990; 1999), Mahadevan (2002a; 2002b); Mahadevan and Kalirajan (2000); Salim (2003; 2008).

Researchers in this field discovered that there are at least two problems with the two-stage approach (Kumbhakar et al., 1991). Firstly, technical efficiency might be correlated with the production inputs, which may cause inconsistent estimates of the production frontier. Secondly, the OLS method in the second stage is inappropriate since technical efficiency is assumed to be one-sided. With these two problems, there is a potential bias in the two-stage approach. Using a Monte Carlo simulation, Wang and Schmidt (2002) show that the bias in the two-stage approach can be very severe.

Aware of these limitations, the recent SFA with exogenous variables then suggests a one-stage approach to overcome these problems.

The one-stage approach is proposed by some scholars. Notably among them are Kumbhakar et al. (1991), Reifschneider and Stevenson (1991), Huang and Liu (1994), Heshmati and Kumbhakar (1994), and Battese and Coelli (1995). The first four papers are conducted in a cross-sectional context, and the last paper is developed in a panel data context. These studies suggest that all parameters are estimates in one-stage in order to obtain consistent estimates.

Similar to the two-stage approach, the technical efficiency in the one-step approach is defined as a function of a set of firm-specific exogenous variables. However, unlike the two-stage method, the parameters of both the production frontier and efficiency effect are estimated simultaneously using a ML method, under appropriate distributional assumptions for both error components (v_i and u_i). For the merit of the one-step approach and for its compatibility with panel data, the present study discusses in more detailed the one-step stochastic frontier model proposed by Battese and Coelli (1995).

The One-Stage Battese and Coelli (1995) Model

The one-stage stochastic frontier model proposed by Battese and Coelli (1995) is similar to equation (14a) for the production frontier and equation (14b) for the inefficiency effect that incorporates exogenous variables.[2] To explain this in more detail, the model is rewritten below in a general functional form

$$Y_{it} = f(\mathbf{X}_{it};\boldsymbol{\beta}).\exp(v_{it} - u_{it}) \qquad (16a)$$

$$u_{it} = \mathbf{z}_{it}\boldsymbol{\delta} + \mu_{it} \qquad (16b)$$

where Y_{it} denotes the scalar output of firm i $(i=1, 2, ..., N)$ at time t $(t=1,2,...,T)$, $\mathbf{X}_{it}$ is a $(1xk)$ vector of inputs used by firm i at time t, $\boldsymbol{\beta}$ is a $(kx1)$ vector of unknown parameters to be estimated; the v_{it} is a random error; u_{it} is the technical inefficiency effect; $\mathbf{z}_{it}$ is a $(1xm)$ vector of observable non-stochastic explanatory variables affecting technical inefficiency for firm i at time t, $\boldsymbol{\delta}$ denotes a $(mx1)$ vector of unknown parameters of the inefficiency effect to be estimated; ω is an unobservable random error.

The underlying assumptions of the above model are:

$$v_{it} \sim \text{iid } N\left(0, \pm_v^2\right) \qquad (17a)$$

$$u_{it} \sim N^+\left(\mathbf{z}_{it}\boldsymbol{\delta}, \pm_u^2\right) \qquad (17b)$$

$$E\left(v_{it}u_{it}\right) = 0 \qquad (17c)$$

$$E\left(\mathbf{X}_{it}u_{it}\right) = 0 \qquad (17d)$$

$$\mu_{it} \sim \mathrm{N}^+\left(0, \pm_u^2\right) \text{, s.t. the point of truncation is } -\mathbf{z}_{it}\boldsymbol{\delta} \qquad (17e)$$

The last assumption implies that the random variable ω_{it} could be negative if $\mathbf{z}_{it}\boldsymbol{\delta} > 0$, i.e. $\mu_{it} \geq -\mathbf{z}_{it}\boldsymbol{\delta}$. As shown by Battese and Coelli (1995), this last assumption is consistent with the assumption (17b).

The parameters of stochastic frontier production function and inefficiency effects in equations (16a) and (16b) are estimated using a ML method. Battese and Coelli (1995) replace the variance of error components $\left(\pm_v^2 \text{ and } \pm_u^2\right)$ with

$$\pm_S^2 = \pm_v^2 + \pm_u^2 \text{ and } ¥ = \frac{\pm_{\mathrm{u}}^2}{\left(\pm_{\mathrm{u}}^2 + \pm_v^2\right)}$$

and obtain the estimated parameters $\left(\hat{\boldsymbol{\beta}}, \hat{\boldsymbol{\delta}}, \hat{\pm}_S^2, \hat{¥}\right)$ from the partial derivation of the log-likelihood function. The detailed derivation of the likelihood function from the density functions of v_{it} and u_{it} is explained in Battese and Coelli (1993). The partial derivatives of the log-likelihood function with respect to the parameters, $\boldsymbol{\beta}, \boldsymbol{\delta}, \pm_S^2$ and ¥ , can be written as:

[2] The Battese and Coelli (1995) model is commonly classified as an extension of random-effect model in the panel data stochastic frontier analysis. An excellent discussion on the classification of panel-data stochastic frontier models into fixed-effect and random-effect is provided in chapter 4 of Kuenzle (2005).

$$\frac{\partial L^*}{\partial \boldsymbol{\beta}} = \sum_{i=1}^{I}\sum_{t=1}^{T}\left\{\frac{\left(y_{it} - \mathbf{x}_{it}\boldsymbol{\beta} + \mathbf{z}_{it}\boldsymbol{\delta}\right)}{\pm_S^2} + \frac{\text{¤}\left(d_{it}^*\right)}{\Phi\left(d_{it}^*\right)}\frac{\yen}{\pm_*}\right\}\mathbf{x}_{it}' \tag{18}$$

$$\frac{\partial L^*}{\partial \boldsymbol{\delta}} = -\sum_{i=1}^{I}\sum_{t=1}^{T}\left\{\frac{\left(y_{it} - \mathbf{x}_{it}\boldsymbol{\beta} + \mathbf{z}_{it}\boldsymbol{\delta}\right)}{\pm_S^2} + \left[\frac{\text{¤}\left(d_{it}^*\right)}{\Phi\left(d_{it}^*\right)}.\frac{1}{\left(\yen \pm_S^2\right)^{1/2}} - \frac{\text{¤}\left(d_{it}^*\right)}{\Phi\left(d_{it}^*\right)}.\frac{\left(1-\yen\right)}{\pm_*}\right]\right\}\mathbf{z}_{it}' \tag{19}$$

$$\frac{\partial L^*}{\partial \pm_S^2} = -\frac{1}{2}\left(\frac{1}{\pm_S^2}\right)\left\{\left(\sum_{i=1}^{I}T_i\right) - \sum_{i=1}^{I}\sum_{t=1}^{T}\left[\frac{\text{¤}\left(d_{it}\right)}{\Phi\left(d_{it}\right)}d_{it} - \frac{\text{¤}\left(d_{it}^*\right)}{\Phi\left(d_{it}^*\right)}d_{it}^*\right] - \sum_{i=1}^{I}\sum_{t=1}^{T}\frac{\left(y_{it} - \mathbf{x}_{it}\boldsymbol{\beta} + \mathbf{z}_{it}\boldsymbol{\delta}\right)}{\pm_S^2}\right\} \tag{20}$$

$$\frac{\partial L^*}{\partial \yen} = \sum_{i=1}^{I}\sum_{t=1}^{T}\left\{\frac{\text{¤}\left(d_{it}\right)}{\Phi\left(d_{it}\right)}\frac{d_{it}}{2\yen} + \frac{\text{¤}\left(d_{it}\right)}{\Phi\left(d_{it}^*\right)}\left[\frac{y_{it} - x_{it} \quad + z_{it}\text{¢}}{\pm_*} + \frac{d_{it}^*\left(1-2\yen\right)}{2\yen\left(1-\yen\right)\pm_*^2}\right]\right\} \tag{21}$$

where L* is a log-likelihood function, $\text{¤}(\bullet)$ represents the density function for the standard normal random variables, $\Phi(\bullet)$ represents the distribution function for the standard normal random variable, T is total period of time, $\pm_* = \frac{\pm_u^2 \pm_v^2}{\left(\pm_u^2 + \pm_v^2\right)}$,

$d_{it} = \frac{\mathbf{z}_{it}\boldsymbol{\delta}}{\left(\yen \pm_S^2\right)^{1/2}}$,

$d_{it}^* = \frac{\left(1-\yen\right)\mathbf{z}_{it}\boldsymbol{\delta} - \yen\left(y_{it} - \mathbf{x}_{it}\boldsymbol{\beta}\right)}{\left[\yen\left(1-\yen\right)\pm_S^2\right]^{1/2}}$, and all other variables are as previously defined.

CONCLUSION

This paper has discussed the framework and the development of Stochastic Frontier Approach (SFA) for measuring efficiency of firms. The original ideas of Aigner *et al.* (1977) and Meeusen and van den Broeck (1977) are presented in the beginning of the paper, to show the basic framework of the SFA. The development of flexible distributional assumptions is then follow. Time-variant technical efficiency models are then developed by some experts to drop the very strong assumption of time-invariant for a production unit. The most recent developed models are the panel data model with time-variant technical efficiency, which allow for estimating the efficiency scores under the two-stage and the one-stage procedures.

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