

Cognitive Load Theory and Mathematics Learning: A Systematic Review

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Abstract. Cognitive load theory (CLT) is an instructional design theory which is based on an information processing view of human cognition. Recently, proponents of this theory have claimed that all forms of “minimally-guided instruction” lead to poorer learning compared to direct instruction because they impose irrelevant cognitive load. This article critically examines this claim by reviewing the empirical evidence that it is based on. This review focuses on studies of mathematics learning. This review concludes that there is little evidence to support the strong claim made by cognitive load theory. Theoretical and practical implications of this finding are discussed.

Keywords: cognitive load theory, mathematics, learning, direct instruction

Abstrak. Teori beban kognitif adalah teori perancangan pembelajaran yang didasarkan pada teori pemrosesan informasi tentang kognisi. Belum lama berselang, beberapa ahli teori beban kognitif menyatakan bahwa semua metode pembelajaran yang mengharuskan siswa untuk memecahkan masalah secara mandiri (minimally-guided instruction) akan membawa hasil belajar yang lebih buruk bila dibandingkan metode-metode pengajaran langsung seperti ceramah atau belajar dari contoh soal. Klaim inilah yang diulas secara kritis dalam artikel ini. Untuk mempersempit permasalahan, ulasan ini hanya akan membahas pembelajaran di bidang matematika. Ulasan ini menemukan sedikit bukti yang mendukung klaim yang dikemukakan para ahli teori beban kognitif. Implikasi teoretis dan praktis dari temuan akan dibahas.

Kata kunci: teori beban kognitif, matematika, belajar, instruksi langsung

This article presents a systematic review of studies of mathematics learning and instruction based on cognitive load theory (Sweller, van Merriënboer, & Paas, 1998; van Merriënboer & Sweller, 2003). This review is set against the background of a recent claim from proponents of CLT that forms of minimally-guided instruction (which the authors take to include “constructivist, discovery, problem-based, experiential, and inquiry-based teaching”) lead to poorer learning compared to direct instruction because they impose irrelevant cognitive load (Kirschner, Sweller, & Clark, 2006).

By extension, Kirschner et al. (2006) are claiming that mathematics instruction should solely consist of direct instruction (e.g. worked examples) and be

stripped of any problem solving and inquiry activities. Clearly, this strong claim bears significant practical implications and hence needs to be carefully assessed. I have chosen to do this with respect to mathematics instruction because, firstly, this is an area which has been central to the development of CLT and as such, it serves as a test bed for Kirschner et al.’s claim. Secondly, mathematics learning has been extensively theorised other theoretical perspectives, including constructivism and socio-cultural theories, which can provide fruitful contrasts to CLT’s own conceptualisation. Before presenting results of this review, this article will outline the main tenets and development of CLT.

Basic Tenets of Cognitive Load Theory

According to Sweller’s (2006) account, CLT originated from research on problem solving during the 1970s. Back then Sweller and his group stumbled upon an intriguing observation: their research par-

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ticipants had difficulties noticing (learning) the solution underlying a set of problems, even after solving many of those problems. This observation came from a series of experiments involving number transformation problems (Sweller, Mawer, & Howe, 1982). In these experiments, undergraduate students had to transform a starting number into a certain goal number. To achieve this goal number, two moves were permitted: multiplying by 3 and subtracting with 69.

Unknown to the participants, all the problems could be solved by alternating the two moves (multiply then subtract) until the goal number is reached. For example, a problem might be to transform “31” into “3”. This can be done by multiplying ($31 \times 3 = 93$), subtracting ($93 - 69 = 24$), multiplying again ($24 \times 3 = 72$), and subtracting again ($72 - 69 = 3$). In the experiments the students used a computer to do the multiplication and subtraction; thus, no mental calculation was needed. Sweller et al. (1982) found that most students solved the problems easily. However, by the end of the experiments, very few of them were aware of the underlying solution (the alternating moves). Thus, solving problems seemed to be a poor way of learning the solutions to those problems. These authors argued that problem solving requires attention to be devoted to the discrepancy between a current state and a goal state (that is, to solve the problem), which means that attention is diverted from attempts to uncover the deep structure or rules which govern a group of problems.

Subsequently, CLT was developed into a more detailed instructional design theory based squarely on an information processing model of human cognition (Mayer, 1992). This model postulates three main structures in the human cognitive architecture: a sensory memory in which information from the environment is first registered; a working memory (WM) which equates to conscious experience; and a long term memory (LTM) which acts as a storehouse of learned information. CLT defines learning as change in the information stored in LTM. For this to occur, new information must be processed in WM before it is stored or integrated in LTM.

Working Memory Limitations and Its Instructional Design Consequence

The core of CLT, I would argue, lies in its description of the instructional consequences of working memory. Citing evidence from cognitive psychol-

ogy (e.g. Miller, 1956), CLT describes WM to be extremely limited both in duration and capacity when handling new information from working memory. Specifically, WM is said to hold about 5 units of new information for only several seconds (Paas, Renkl, & Sweller, 2003; Sweller et al., 1998).

More recently, Sweller (2004) drew analogy from evolutionary theory to argue that to learn (that is, to store new information in LTM), the WM has no choice but to randomly combine new information elements and then test their viability. Because most educational situations mainly expect students to acquire knowledge (stored in LTM) from new information, instruction must take into account WM's severe processing limitations. In specifying the instructional design implications of WM's limitations, CLT postulates three different cognitive loads which are processed by WM: extraneous (which is learning-irrelevant load posed solely by the design of the learning material), intrinsic (which stems from the intrinsic complexity of the material), and germane loads (which directly leads to learning). From a CLT perspective, instructional design basically needs to minimise extraneous load, manage intrinsic load, and promote germane

Much of the earlier CLT literature has focused on reducing extraneous load to free up WM's limited capacity¹ (Chandler & Sweller, 1991; Sweller, 1988; Sweller & Cooper, 1985; Ward & Sweller, 1990). It was this principle of minimising extraneous load which led proponents of CLT to advocate against problem solving and inquiry activities (Kirschner et al., 2006; Sweller, Kirschner & Clark, 2007). It was also this principle which brought the discovery of novel instructional design effects, including the worked

attention effect, the redundancy effect, and the modality effect. (Readers are referred to other articles: Sweller et al., 1998; van Merriënboer & Sweller, 2003 for a more detailed description of these effects.)

Schemas, Long term Memory and the Expertise Reversal Effect

CLT also alludes to the nature of LTM when describing the human cognitive architecture, but this reference is less consequential (compared

¹The management of intrinsic load and the promotion of germane load have only more recently become a focus, as reflected in the studies of mathematics learning reviewed below.

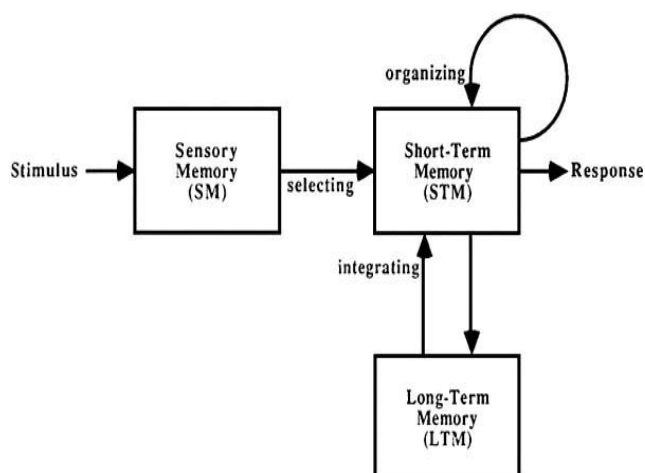


Figure 1. The human cognitive structure and processes, as described by Mayer (1992, p. 408)

to the nature of WM) in specifying instructional design principles. Theoretical writings of CLT often cite findings from expertise research, especially chess expertise (e.g. Chase & Simon, 1973), to show the importance of LTM (Sweller et al., 1998; van Merriënboer & Sweller, 2003). Specifically, information stored in LTM is described in terms of schemas, which are “cognitive structures that enable problem solvers to recognise problems as belonging to particular categories requiring particular operations to reach a solution.” (Paas & van Merriënboer, 1994, p. 123). It is the presence or absence of such schemas that explain the differences between levels of expertise. Thus, having those schemas, “[chess] experts may both recognise most configurations met and know which move is most appropriate in that situation” while “novices must rely entirely on general search heuristics” (Sweller & Cooper, 1985, p. 60).

Schemas from LTM can be brought to WM during learning. Information which is subsumed by a schema can be processed as one chunk by WM. In other words, WM has much larger capacity when dealing with information that can be recognised or parsed by available schemas. This means that the intrinsic load (i.e. the complexity or level of difficulty) of a learning material is not fixed, but depends also on a learner’s available schemas (i.e. his/her level of expertise in that domain).

So far, CLT has pointed out only one instructional design effect of the interaction between WM

and LTM: the expertise reversal effect. This refers to the finding that direct instructions (e.g. worked examples) become less effective, while problem solving activities become more effective, as learners become more experienced in a domain (Ayres, Chandler, Kalyuga, & Sweller, 2003; Chandler, Kalyuga, Sweller, & Tuovinen, 2001). CLT explains that this effect occurs because experienced learners already have domain-relevant schemas which enable them to process many chunks of information as a single element. Consequently, such learners should have enough WM capacity to learn from problem solving. Conversely, worked examples become less effective because they are redundant (they present information already available in the learners’ schemas).

To summarise, the basic tenets of CLT can be schematically represented in Figure 1 above. Instructional design determines the learner’s cognitive activity in WM, which can be either relevant (germane load) or irrelevant (extraneous load) to learning. Expertise and the complexity of a material determine the difficulty or intrinsic load. If extraneous load is low, then WM has the capacity to engage in germane load. If germane load is sufficient to tackle intrinsic load, then schema acquisition and automation become possible.

Research Question and Method

The main question addressed is: To what extent is the claim that ‘direct instruction is superior to minimally-guided instruction’ empirically supported by studies of mathematics learning and instruction conducted within the framework of CLT? To address this question, a systematic review was performed by searching the GoogleScholar database using the keywords “cognitive load theory” and “mathematics instruction”. Titles and abstracts obtained from this search were read. Only articles reporting empirical studies of mathematics learning which explicitly used cognitive load theory as their framework were selected for the present review. This procedure resulted in eleven studies, some of which comprised of multiple experiments.

This review extracted information on the following aspects of each selected study: 1) study participants, 2) area of mathematics, 3) measurement of learning outcomes, and 4) results related to each mea-

Table 1
Studies Related to Reducing Extraneous Load

Study	Participants and area of math	Measurement of learning outcome	Results related to learning outcome
(Sweller & Cooper, 1985): Exp. 2 to 5	9 th grade students studying algebra (equation manipulation (e.g. For equation " $a = ag + b$ ", express a in terms of other variables").	Near transfer on isomorphic and modified items.	Worked examples superior to conventional problem for similar items, but NOT for modified items.
(Paas, 1992)	2 nd year technical school students (aged 16-18) studying statistics (calculating central tendency measures).	Near transfer on isomorphic and modified items.	Worked examples superior to conventional problem for similar and modified items.
(Paas & van Merriënboer, 1994)	4 th year technical school students (age 19-23) studying geometry (right-angled triangles, Pythagorean theory, and trigonometry).	Near transfer on isomorphic items.	Worked example superior to conventional problem.
(Mousavi, Low, & Sweller, 1995): Exp. 1 to 4	8 th grade students studying geometry (relations between congruent triangles & circles; & relations between congruent triangles & squares)	Near transfer on equivalent and isomorphic items.	Presentation format (to use dual modality and reduce split attention) did NOT improve learning outcome.
(Mwangi & Sweller, 1998): Exp. 1 & 2	3 rd and 4 th grade (primary) students studying arithmetic ("compare" word problems: "more than" and "less than").	Near transfer (with 10 days delay) on equivalent and isomorphic items.	Worked examples NOT superior to conventional problems. Integrated worked example superior to split attention worked example for equivalent, but NOT isomorphic, items.
(Atkinson, 2002)	Undergraduate students studying arithmetic (proportional reasoning e.g. If a person is 6-feet tall and has a 9-feet shadow, how tall is a building which has a 90- feet shadow?).	Near transfer on isomorphic and adaptive items.	Dual modality example superior to single modality worked example for equivalent, but NOT isomorphic, items.

sure of learning outcome. Information on these aspects were then analysed to shed light on whether mathematics instruction should consist of solely direct instruction.

Findings

The studies of mathematics learning reviewed here cover the three instructional design approaches which CLT conceptualised: reducing extraneous load, mana-

ging intrinsic load, and promoting germane load. All studies were controlled or randomised experiments, with the interventions administered mostly individually, but occasionally also in group settings. Most interventions were held in short, single sessions, except in one study which took place across three sessions. The learning effects of the interventions were measured by tests of transfer conducted by the experimenters, in the same location, usually right after the intervention and using the same format (e.g. written

Table 2
Studies Related to Managing Intrinsic Load

Study	Participants and area of math	Measurement of learning outcome	Results related to learning outcome
(Clarke et al., 2005)	9 th grade students ($N = 24$) studying geometry (linear & quadratic functions and their graphical representations).	Near transfer on equivalent, isomorphic, and modified items.	Students with low prior knowledge of spreadsheet learned better with a sequenced material (spreadsheet first, then math concepts).
(Gerjets, Scheiter, & Catrambone, 2006)	University students (age around 24-25) studying probability of complex events (calculating permutations and combinations).	Near transfer on isomorphic and modified items.	Modular examples (which break down examples into sub-problems) led to better learning outcome.
(Ayres, 2006)	8 th grade students (age 13) studying algebra (opening bracketed equations e.g. $5(3x-4) - 2(4x)$ etc.).	Near transfer on equivalent and isomorphic items.	In Exp. 1, isolated examples (which present single solution steps one at a time) led to fewer errors. In Exp. 2, isolated examples did NOT lead to fewer errors for students with low prior knowledge, and instead led to MORE errors for students with high prior knowledge.

or computer-based) as used in the learning phase. Using Barnett and Ceci's (2002) transfer taxonomy, these CLT studies can be regarded as using "near" tests of transfer. (Issues of learning and transfer will be examined in more detail later).

Table 1 summarises six studies related to reducing extraneous load. These studies cover arithmetic, algebra, geometry, and statistics at the primary, secondary, and tertiary levels. In these studies, extraneous load was reduced by using worked examples (compared to conventional problems) and by structuring worked examples to reduce split attention and utilise WM's dual modality (visual and auditory). The results, with respect to learning outcomes, present a rather mixed picture of the worked example, split attention, and dual modality effects. For example, while some studies clearly show the superiority of worked examples (Paas, 1992; Paas & van Merriënboer, 1994), other studies were less clear (Mwangi & Sweller, 1998, Experiment 1; Sweller & Cooper, 1985). In

Experiment 3 of Mwangi and Sweller, the integrated worked example (designed to reduce extraneous load by not demanding learners to mentally integrate information) did not yield better learning outcome.

Studies related to the management of intrinsic load presented in Table 2. These three studies cover algebra, geometry, and probability concepts in secondary and tertiary levels. In one study (Clarke, Ayres, & Sweller, 2005), which involved materials that can be logically separated (spreadsheet and geometry be concepts), intrinsic load was managed by sequencing the materials into different learning sessions (study spreadsheet first, then geometry). In the other studies, intrinsic load was managed by presenting worked examples which were broken down into sub-problems or separate steps. The results mostly showed that the interventions produced better learning outcomes (with one exception: Experiment 2 of Ayres (2006). In addition, these studies demonstrate the expertise reversal effect.

Table 3
Studies Related to Promoting Germane Load

Study	Participants and area of math	Measurement of learning outcome	Results related to learning outcome
(Paas & van Merriënboer, 1994)	4 th year technical school students (age 19-23) studying geometry (right-angled triangles, Pythagorean theory, and trigonometry).	Near transfer on isomorphic items.	High-variability worked examples were superior to low-variability worked examples.
(Gerjets et al., 2006)	University students (age around 24-25) studying probability of complex events (calculating permutations and combinations).	Near transfer on isomorphic and modified items.	<p>Worked examples supported with direct explanation were NOT superior to those without support.</p> <p>Worked examples supported with self explanation prompts led to WORSE outcomes compared to examples without the prompts.</p>
(Grobe & Renkl, 2006)	University students (age around 21-22) studying combinatorics (counting permutations & combinations) and probability (calculating the probability of certain events).	Near transfer on isomorphic and modified items.	<p>Worked examples with high variability (multiple solutions) were superior to examples with low variability (single solutions).</p> <p>Worked examples supported with self explanations were NOT superior to examples without the support.</p>
(Mwangi & Sweller, 1998): Exp. 1 & 2	3 rd and 4 th grade (primary) students studying arithmetic (“compare” word problems: “more than” and “less than”).	Near transfer (with 10 days delay) on equivalent and isomorphic items.	Worked examples supported with self explanations were NOT superior to examples without the support.

Studies which attempt to promote germane load are described in Table 3 (some studies, which contain multiple or factorial experiments, have been listed above). In One way to promote germane load was by increasing the variability of worked examples. The rationale for this came from research on analogical reasoning: examples which vary terms of their surface structure (e.g. their values, cover stories, or solution

steps) should direct attention to the invariant deep structure of those problems. The two CLT studies below show that this is the case. Another way to promote germane load was by giving direct explanations, or prompting self-explanations of the worked examples. The rationale for this is derived from research on self explanations, which show that, when studying worked examples, “good” learners elabo-

rate the missing or unexplained steps (Chi, Bassok, Lewis, Reimann, & Glasser, 1989). However, the CLT studies that use this technique did not succeed to improve learning outcomes.

Discussion

The Conceptualisation of Learning and Transfer in CLT

As noted above, CLT describes learning as the acquisition and automation of schemas, are often described as cognitive structures representing problem categories and their solution procedures. This view is clearly reflected in two aspects of the mathematics instruction studies above: the choice of tasks or learning objectives, and how learning outcome is measured. The studies typically do not elaborate in detail any learning objectives of the interventions.

Nonetheless, it seems that the learning objectives are quite constrained and focus on the acquisition of mathematically correct problem solving procedures. For instance, in Ayres' (2006) study, the objective was to learn how to open bracketed equations such as " $5(3x-4) - 2(4x)$ ". In Paas' (1992) study, students studying about central tendency were "expected to calculate the arithmetic mean, median and mode" (p.430). In other words, the learning objectives are defined in terms of an underlying "deep structure" which can be regarded as the essence of a problem category and its relevant solution moves. Indeed, this is embodied in the concept of worked examples, which are designed to present the essence of problems and their solution procedures.

With regards to measurement of learning outcome,

the CLT studies reviewed here used tests of recall and transfer. These tests reflect CLT's schema acquisition view of learning and can be described using Reed's (1993) schema-based theory of transfer. Recall tests consist of items identical in terms of deep structure and also surface structure (e.g. values, wording, cover stories) to the items used during learning phase. Transfer tests consist of items which share the same deep structure, but presented in different surface structures.

Transfer test items can be further categorised into "isomorphic" and "modified" items (Reed, 1993). Isomorphic items are problems that have different surface structures but can be solved using the exact same procedure to the items used in the learning phase. Modified items are problems that have different surface structure and a modified deep structure, which means that they have to be solved using a modification of the learned procedures. For example, in Paas' study, the isomorphic items (termed "near-transfer" problems) require students to use the same arithmetic formula to calculate the same value as practiced before. In contrast, the modified items (termed "far-transfer" problems) demanded students to use a formula to calculate a different value (but from the same formula) to the one practiced (see Figure 2).

The nature of tasks, learning objective, and transfer tests in the CLT studies reviewed here belie the assumption that instruction equates to exposing students to the essence or deep structure of problem types. Learning, then, equates to discovering these deep structures; and knowing is having knowledge in the form of abstract, decontextualised mental structures or schemas. The purpose of education research becomes finding the most efficient way of helping students to do this (i.e. in CLT, this basically translates to finding the best ways to structure and present worked exam-

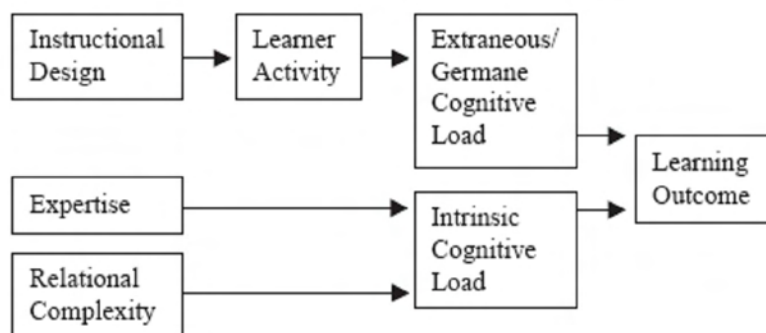


Figure 2. Schematic representations of CLT's basic tenets (Gerjets & Scheiter, 2003, p. 35)

ples). A corollary assumption is that language is regarded as a vehicle to transmit, from a teacher or instructional material to students, the meaning or the essence of problems and their solutions. (This is clearly reflected in Sweller's (2004, p. 18) description that knowledge in a teacher's LTM can be transmitted through communication to help organise students' processing of novel information).

This set of assumptions makes CLT fall in the "nature as template/instruction by nature" (and far from the "humans as creators") end of the Phillips' (1995, p. 8) defining dimension of constructivist thought. Nature or reality is seen as objective in the sense that it is separate from, but ultimately known by, the individual subject. Knowing or learning means discovering patterns of this objective reality (which include mathematical reality). In other words, CLT reflects a realist epistemology or representational view of human mind, which is squarely at odds with the non-dualist epistemology of constructivism (Cobb, Yackel, & Wood, 1992) and situated learning theories (Lave, 1988; Lerman, 1999). Hence, although Kirschner et al. state that "the constructivist description of learning is accurate" (i.e. that knowledge is constructed by learners), their position is fundamentally different from most constructivist and situated learning theorists. I now turn to these theories to analyse the CLT studies' transfer tests and their results.

Analysis of CLT Studies' Transfer Tests

As noted previously, the transfer tests can be re-

garded as tests of near transfer in every dimension in Barnett and Ceci's (2002) taxonomy. No test of far transfer (tests across longer time span, in different locations, administered by different people and perhaps within a different social context) has been conducted. I argue that the lack of far transfer is not a coincidence or only due to practical difficulties. Rather, this may indicate CLT's realist epistemology and schema-based view of learning and transfer. When learning is seen as acquiring abstract, decontextualised schemas, transfer should be unproblematic as long as the relevant schemas are available in LTM. Being abstract and unrelated to their context of acquisition, schemas should be easily "summoned" when needed in future situations. There is no qualitative difference between near and far transfer. If one can demonstrate the occurrence of near transfer, then far transfer should necessarily occur. The consequence of this view for research is that there is no need to conduct tests of far transfer!

This position, however, is no longer tenable in light of evidence of the disjunction between mathematics performances in one social context to another (overviewed by Lerman, 1999). For instance, Carraher and colleagues (1985) investigated the mathematical strategies and performance of several Brazilian children in different contexts (selling goods as street vendors and solving word problems as students). Although the problems were mathematically identical, the strategies used differed from one context to another. Performance was also better for problems embedded in the children's "real-life" context (selling goods as

The table below shows the temperature (Celsius) at 3:00 PM on 5 days of July in the city of Enschede.

Day	1	2	3	4	5
Temperature	18	16	20	21	19

Calculate the mean temperature for this period.

Calculate the mean for the following row of numbers:

13, 16, 16, 13, 14, 12, 15, 25

What number is presented by X in the frequency table below, when it is known that the mean equals 5?

Number	Frequency
3	1
4	4
5	5
6	2
7	X

Figure 3. Example of "isomorphic" (middle box) and "modified" (right box) transfer items in Paas' (1992, p. 431) study.

street vendors). Similar findings were reported by Lave (1988), who investigated people's mathematics performance in the context of shopping and school problems.

The lack of cross-context transfer is difficult to explain from CLT's perspective. According to CLT's schema-based theory, if a person can solve arithmetic problems in the context of buying/selling goods, this is because she has acquired the schemas for the relevant problem types and solutions. As such, she should have little problem in applying those schemas for arithmetically equivalent problems in other contexts (such as word problems in a classroom test).

To better explain data on cross-context transfer, theorists of situated cognition/learning postulate that knowledge is not abstract but at least partly, if not fundamentally, context specific. In Lave's (1988, p.1) words, cognition is "distributed—stretched over, not divided among—mind, body, activity, and culturally organized settings (which include other actors)." Learning is described as enculturation into the practices of a community, practices which are mediated by cultural tools such as mathematical concepts and algorithms (Brown, Collins, & Duguid, 1989). These cultural tools cannot be divorced from their function in practices deemed valuable by a community. Hence, knowledge is inherently social. At the social level, people learn to become more competent in performing practices and to become more central members of a community (Lave & Wenger, 1991). At the psychological level, this process of learning involves constructing increasingly viable knowledge (viable in terms of a community's practices) by drawing on the community's available cultural tools (Cobb, 1994).

From the sociocultural perspective, transfer has been described in terms of "consequential transitions" from participation in one social practice to another (Beach, 1999). Such transitions involve multiple processes such as transformation of knowledge, skills, emotions, tacit values, and identities. Transfer cannot be explained solely by decontextualised mediational means (i.e. abstract schemas). It is not surprising, from this perspective, to find disjunctions between knowledge learned from school and performance in other contexts: Mathematical practice in classrooms (the culture of word problems, of paper and pencil tests, of academic competition, of a student's acceptable behaviour, etc.) is fundamentally different to the mathematical practices in other contexts.

The transfer tests used in CLT studies above capture transitions within a single social practice (the classroom or experimental activity). The lack of evidence, or even attempts to show transfer across practices places serious limitations to the usefulness of the instructional design principles (in relation to mathematics) stipulated by CLT. This is exacerbated with less than unequivocal results of the studies' near transfer tests, in particular for problems which require students to modify the learned procedures/algorithms. This clearly undermines CLT's assertion that mathematics instruction should consist solely of worked examples, however well designed they are.

Conclusion

CLT describes learning as change in LTM, or more specifically, the acquisition of new schemas in LTM. To be stored as schemas in LTM, novel information must be processed in the WM. Given the limitation of WM (for processing novel information), learning cannot occur when WM is burdened with unnecessary load, such as means-end analysis imposed by problem solving and inquiry activities. Hence, proponents of CLT have advocated against instructions which emphasise problem solving and inquiry activities (Kirschner et al., 2006; Sweller et al., 2007). Indeed, for these authors, instruction should solely consist of well designed direct instruction which exposes learners to the complete set of information they are to acquire.

Kirschner et al.'s recommendation logically extends to mathematics learning and instruction. But to what extent do results of these studies support the elimination of problem solving and inquiry activities? I argue that the instructional objectives reflected in CLT mathematics studies are overly narrow, focusing mainly on acquisition of algorithms or computational abilities. I believe that this reflects a realist view that depicts learning/knowning as discovering patterns of an objective reality. This realist position is reflected in the use of worked examples to expose learners to deep structure of mathematics problems, with the goal of helping students to acquire schemas of problem types and their relevant algorithms. Given the lack of far (cross-context) transfer evidence, along with the mixed evidence on near transfer, the ultimate usefulness of such narrow instructional objective (the acquisition of abstract schemas) is seriously limited.

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